Automated prediction of loudness growth curve using EEG signals

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***Abstract*—This paper introduces an innovative and automated approach for the prediction of loudness growth curves based on auditory brainstem responses (ABRs), harnessing the power of deep learning and signal processing techniques. Hearing loss, affecting a significant portion of the global population, calls for accurate and efficient assessment methods to improve the quality of life for affected individuals. Our method entails preprocessing ABR signals, extracting informative features via empirical wavelet transform with Fourier Bessel series expansion, and subsequently mapping these features to loudness growth estimates using multi-target regression. Through evaluation employing mean squared error and Frechet distance, our approach demonstrates acceptable performance and consistency across subjects and stimulus levels. Importantly, it overcomes limitations inherent in existing methods that primarily rely on click ABRs and psychoacoustic measures.**

I. Introduction

According to findings in [1] based on the Global Burden of Disease Study 2019, every one person in five has hearing loss. It can be congenital or may occur due to aging or exposure to loud noise for prolonged periods. Hearing loss if left undiagnosed and unattended adversely affects the quality of life. It may hinder speech development in children and can cause loneliness, isolation, depression, dementia, and cognitive decline in the elderly [1]. Therefore, there is a need for timely hearing screening tests followed by a prescription of appropriate assistive hearing devices such as cochlear implants or hearing aids. As the hearing loss varies in nature and intensity, comprehensive hearing screening tests are needed to assess the loss type and severity. The commonly used procedures for assessment of auditory function include air conduction and bone conduction pure-tone thresholds, speech recognition thresholds, suprathreshold speech recognition, etc. [2]. However, speech-based audiometric procedures have been standardized for English-speaking adults and therefore not suitable for hearing assessment of non-native English speakers. Further, these tests require active participation and feedback from the listeners during assessment which may be difficult for children and infants.

Other methods of auditory function assessment include otoacoustic emissions (OAEs) and auditory evoked potentials (AEPs). They are useful for the assessment of infants and young children and may be performed in addition to routine audiometric measures for adults. AEP-based hearing threshold estimation makes the estimation language independent and is suitable for infants, children, and persons who lack the ability to provide a reliable response to the stimuli. Perceived loudness is a subjective quantity and is correlated to the physical sound intensity. The human perception of loudness is a function of stimulus intensity, stimulus frequency, and depends on individual listener. It varies in complex ways that cannot be accounted for by the listener's threshold level. For example, in listeners with sensorineural hearing loss the hearing thresholds are elevated, and the perceived loudness increases rapidly as compared to normal hearing listeners. Therefore, there is a need for estimating the loudness growth along with hearing thresholds and discomfort levels from physiological signals like AEP or auditory brainstem response (ABR) for normal as well as hearing-impaired listeners.

Numerous mathematical models have been developed to estimate loudness growth as a function of stimulus intensity, with early methods focusing on non-linear models for loudness prediction. An initial model proposed a power law relationship between loudness L (in Sones) and sound pressure level P (in Pascals) as $L = kP^{0.6}$, which is accurate for higher sound pressure levels relative to hearing threshold levels, was proposed in [3]. Hellman and Zwislocki [4] refined this model to $L = 0.01(P - P_0)^{0.6}$, where P_0 is a constant. Buus and Florentine [5] further advanced the model by incorporating the effect of Signal-to-Noise Ratio (SNR) using $L = k10^{s_{HL}\frac{HL}{10}}\left[\left(1+\left(snr_{th}.10^{\frac{SL}{10}}\right)^{s_{lo}}\right)^{\frac{s_{hi}}{s_{lo}}}-1\right]$, where snr_{th}, s_{lo}, and s_{hi} are free parameters and k is a scale factor that does not affect the predictions of the model. In this expression, s_{HL} determines the extent to which the loudness at threshold varies with the amount of hearing loss, snr_{th} may be considered the signal-to-noise ratio at threshold (reported as $\mathrm{SNR}_{th} = 10\log(snr_{th})$ dB), s_{lo} is the asymptotic exponent of the loudness function at low levels, and s_{hi} is the asymptotic exponent at high levels. Both the functions proposed are suitable for use at higher stimulus. Further the exponent factor is very different for different loudness levels. Particularly, the loudness growth exponent was larger than 2 for stimulus levels closer to the threshold and was smaller than 0.6 for moderate and higher stimulus levels.

The procedures to measure loudness typically involve psychoacoustical methods such as Cross Modality Matching (CMM) and Magnitude Estimation (ME), which rely on patient responses and can yield inconsistent judgments [6]. Consequently, researchers have explored objective loudness estimation using auditory brainstem responses (ABRs) [6], [7]. Studies have correlated ABR features, especially Wave V latency and amplitude, with psychoacoustical measures of loudness growth. Pratt and Sohmer (1977) [8] utilized AME and assessed ABR features including latency, amplitude, and area of waves (I-V), finding correlations via power function fitting but noting variability across subjects and sessions. Davidson et al. ([9]) also explored Wave V correlations with AME, using Spearman's rank correlation coefficient across stimulus levels, revealing correlations mainly across sessions. In contrast, [10] identified Wave V latency correlations with loudness in normal listeners but not in those with sloping hearing loss. Objective evaluation of loudness discomfort levels has been attempted by correlating Wave V features with subjective assessments [11], [12].

Epstein Silva ([6], [7]) employed tone burst ABRs with AME and CMM measures for objective loudness growth estimation, employing MSE as an evaluation metric rather than Pearson correlation coefficient used in previous studies. Hoseingholizade [13] investigated chirp-evoked ABRs, averaging trials using Bayesian-informed weighted averaging and fitting linear or power functions to estimate loudness growth based on peak to trough amplitudes at different intensity levels.

Significant gaps persist in current research on loudness curve estimation using auditory brainstem responses (ABRs). Earlier methods have primarily relied on click ABRs, potentially limiting robust analysis of loudness curves. Moreover, existing methodologies heavily favor psychoacoustical measures like Cross Modality Matching (CMM) and Magnitude Estimation (ME), often overlooking other valuable ABR data aspects. This bias restricts the full potential of ABR signals for comprehensive feature extraction, typically focusing subjectively on parameters such as amplitude and latency. Furthermore, the field has predominantly used polynomial and exponential functions for loudness growth estimation, neglecting exploration into advanced methods like deep learning and signal processing for ABR data. Despite an extensive literature search, recent studies on automated prediction of loudness growth curves using EEG signals were notably absent, underscoring a critical research gap. Our study addresses these limitations by proposing a novel neural network-based approach, highlighting the need for further research to enhance ABR-based loudness curve estimation capabilities.

This paper introduces a novel approach employing deep learning for automated estimation of the loudness growth curves from the raw ABR signals. Moreover, we explore feature extraction technique, drawing from both signal processing and deep learning methodologies, to predict loudness growth curves. The proposed methodology encompasses several key steps: i) Pre-processing of ABR signals and loudness growth data, ii) Extracting features from ABR data through a signal processing approach called empirical wavelet transform with band boundaries determined by Fourier Bessel series expansion (EWT-FBSE), and iii) Mapping the extracted EWT-FBSE features to corresponding psychoacoustical estimates of loudness growth utilizing Multi-target Regression. We evaluating the efficacy of the proposed approach through objective measures such as mean squared error and Frechet distance and use the Physiobank database [14] for training and testing the proposed model.

II. Methodology

The steps involved in estimation of the relationship between the raw ABR signals and loudness growth curve are explained in the following subsections.

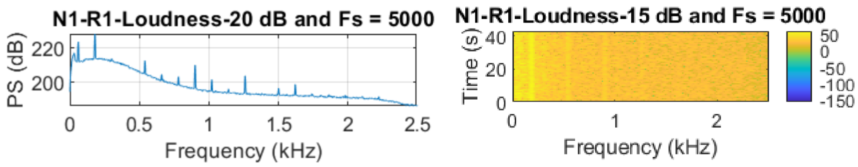
A. Pre-processing of ABR signals and loudness growth data

The processing of the raw files present in the Physiobank "Evoked Auditory Responses in Normals across Stimulus Level" database [14]. It involves extraction of the ABR signal and its metadata using functions from the WaveForm DataBase (WFDB) Toolbox from MATLAB. The database consists of two sets of signals - ABR and OAE and two sets of psychoacoustical estimates of loudness as a function of peak sound pressure level (peSPL)- Cross Modality Matching (CMM) and Magnitude Estimation (ME). There are 2002 samples per trial for 8 normal hearing listeners (four males, four females) with ages 19-31 with audiometric thresholds not exceeding 15 dB HL at octave frequencies 250 Hz - 2 kHz.

The raw ABR signal is averaged over the first 1000 trials for each loudness level to reduce the effect of noise in analysis. The stimuli are originally sampled at 48 kHz and they resampled to 2 kHz, 3 kHz, 5 kHz and 10 kHz. The power spectrum and spectrogram as shown in Figure 1 are used to examine the signal's frequency content. It can be seen that the frequency content is in the range of 0 – 2.5 kHz and in the power spectrum plots, peaks are primarily observed in the 0 - 1.5 kHz range. Thus, for further analysis, the sampling frequency is taken as 3 kHz.

B. Feature extraction

For feature extraction, we begin by decomposing each raw ABR signal using wavelet transform. Wavelet transform, known for its multiresolution analysis properties, is chosen as a suitable method for signal analysis. Empirical wavelet transform (EWT) is specifically employed to decompose the pre-processed signals into a set of intrinsic modes, effectively separating dominant modes from irrelevant ones. This approach has proven to be highly valuable, particularly when analyzing non-stationary and noisy signals. However, it's worth noting that EWT may not achieve an accurate representation of frequency compo nents in signals with shorter durations. Additionally, when frequency components are closely spaced, EWT may not be as effective. To address these limitations, we employ Fourier Bessel series expansion (FBSE) to compute band boundaries within the EWT framework. This combination



(a) a) Power spectrum

(b) b) Spectrogram

Fig. 1: a) Power spectrum and b) spectrogram for averaged ABR signal sampled at 5 kHz.

of EWT and FBSE allows us to overcome these challenges, ensuring a more robust and accurate decomposition of the ABR signals for subsequent analysis. The computation of EWT-FBSE is done using the following procedure in [15]:

$$S(i) = \frac{2}{N^2 J_1(\tau_i)]^2} [\sum_{n=0}^{N-1} ns(n) J_0\left(\frac{\tau_i n}{N}\right) i = 1, 2, 3....N \quad (1)$$

where $J_0(.)$ and $J_1(.)$ are 0th and 1st order Bessel functions, respectively. The N positive roots of $J_0(\tau_k) = 0$ are τ_i for $i = 1, 2, \ldots, N$. The frequency of signal f_i (Hz) and τ_i are related mathematically as $\tau_i \approx \frac{2\pi f_i N}{F_s}$, where F_s represents a sampling rate of the signal. For designing the filter banks based on the FBSE-EWT method, the obtained FBSE spectrum was segmented based on i-th order range boundaries using the scale-space boundary detection method [16]. First (ω_1) and last (ω_N) boundary frequency values are 0 and π, respectively. The optimal set of boundary frequencies for adaptive segments are denoted as ω_i. The empirical scaling and wavelet functions are applied to design the band-pass filters on each adaptive segment of the FBSE spectrum. Wavelet-based band-pass filters were constructed based on [17].

C. Mapping the features to psychoacoustical estimate of loudness growth

The next step is mapping the extracted features from ABR signals to the corresponding psychoacoustical estimate of loudness growth. CMM is taken as the measure of the loudness growth (ranging from -1 to 2). This step involves mapping the features from ABR signals for all stimulus levels to the corresponding loudness growth curve for a given individual. This is achieved using multi-target regression (MTR) based convolutional neural network (CNN) that helps in learning the correlation between the features from ABR signals for different stimulus levels and the loudness growth curve. For prediction of the entire loudness growth curve in one go, MTR is beneficial as it facilitates simultaneous prediction of multiple continuous variables from a set of input variables. The output of MTR is given to a dense layer to obtain the final loudness growth curve for a listener. A deep hard sharing-based network is used for MTR which is connected to a fully dense layer with a linear activation in the third and final stage to predict the loudness growth curve with the number of outputs equal to the number of loudness levels of stimuli present in the input EWT data. The proposed architecture is shown in Figure 2.

The Multi-Target Regression Block in our methodology employs a CNN that captures both spatial and temporal characteristics within datasets. In our specific context, the extracted features are essentially two-dimensional, as they pertain to different stimulus levels and for each level it has 10-band EWT-FBSE coefficients. The CNN architecture consists of a convolutional layer, a max-pooling layer, and a dropout layer, which collectively capture the spatial dependencies within the data. The convolutional layer utilizes 32 filters of size 3x3, effectively discerning spatial patterns with padding set to "same" and a stride of 1. The dropout layer randomly omits a fraction of connections from the preceding layer, promoting model generalization. Batch Normalization is applied after each convolutional block to normalize the extracted features.

In a general MTR framework, inputs are represented as continuous variables. For an input vector X of dimension d, the output space has a dimension q. Each multi-target example, denoted as X_i and Y_i, defines the input-output relationship between vectors of dimensions d and q, respectively. In this supervised deep learning approach, the deep forward network incorporates shared layers where parameters are common among different input variables. The advantage of using a shared DNN architecture lies in its ability to reduce the number of parameters compared to separate networks for each band, thus enhancing computational efficiency and potentially improving generalization by leveraging shared knowledge across different inputs. The shared architecture is followed by non-shared layers, leading to the ultimate target output vector. In our implementation, we construct a shared deep neural network wherein the band outputs serve as features inputted into a shared CNN. The CNN's output is then connected to a fully connected dense layer, facilitating the prediction of the final set of loudness growth estimates using CMM. In essence, this process yields the loudness growth curve tailored to the individual under consideration.

III. Architecture Details of MTR CNN Model

In the implementation of the proposed architecture, a shared deep neural network is developed where the band outputs are first fed into the neural net as features in a shared CNN. The output of the shared CNN is then connected to a fully connected dense layer to predict the final set of loudness growth CMM estimates, i.e., the loudness growth curve for the given individual. The architecture details are given in Table I.

IV. Experiments and Evaluation

A. Model hyperparameters, loss function, and platform

For training the proposed model, we partitioned the dataset into training and testing subsets, following an 85-15 ratio. For the training process, we employed the Adam optimizer. The

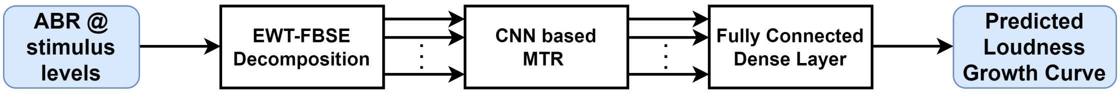


Fig. 2: Prediction of loudness growth using proposed approach.

TABLE I: Architecture Details of MTR CNN Model

Layer Type	Output Shape	Number of Parameters
Input	12, 10, 126	0
Conv 2D-1	12, 10, 32	36,320
Conv 2D-2	4, 3, 32	9,248
Conv 2D-3	2, 2, 32	9,248
Final Dense	12	1,548
Total Parameters		61,484

choice of Mean Squared Error (MSE) as the loss function is motivated by its superior performance in capturing the correlation between the output and non-linear features, as opposed to other loss functions such as Mean Absolute Error (MAE) and Pearson Correlation. To ensure that the validation loss remained under control during training, we implemented an early stopping mechanism with a patience parameter set to 40. Our model underwent training for a maximum of 500 epochs, utilizing two Nvidia RTX A4000 GPUs, each equipped with 16 GB of memory. The learning rate was set at 0.0001, and we used a batch size of 12 to optimize the training process efficiently.

B. Evaluation metrics

For evaluating the performance of our model, we employ two key metrics. The first metric is the Average Mean Squared Error, which quantifies the disparity between the CMM loudness growth estimate and the predicted loudness growth estimate. This metric serves as a crucial performance measure, gauging the accuracy of our predictions. Additionally, we utilize the Frechet distance [18] to assess the similarity between two curves. In our specific context, the Frechet distance is employed to measure the likeness between the predicted loudness growth curve and the ground truth loudness growth curve. This metric is defined as the smallest value among the maximum pairwise distances between the two curves. Notably, we utilize the discrete Frechet distance variant due to the discrete nature of our data points.

V. Results

In this study, we aimed to identify the optimal model for predicting a generalized loudness growth curve. To achieve this, we modified our approach by leveraging a dataset encompassing seven subjects. The dataset includes detailed measurements of auditory responses, allowing us to train and evaluate our models effectively.

To ensure a robust evaluation, we adopted a cross-validation strategy. In each iteration of this strategy, we set aside one subject's data as the test set while using the remaining subjects' data for training the model. This leave-one-out approach allowed us to systematically predict the loudness growth curve for each subject and subsequently compare the predictions to the ground truth curves. This method not only ensures that each subject is tested independently but also helps in understanding how well the model generalizes to unseen data.

To provide a comprehensive view of the model performance, we present a comparative analysis in Table II. The table includes the average Mean Squared Error (MSE) and standard deviation for models developed using K-Fold resampling to predict the loudness growth curve across all subjects. In this context, K-Fold resampling ensures that the training and testing processes are repeated multiple times, enhancing the reliability of the performance metrics. The columns in the table contain the average MSE (standard deviation) scores across stimulus intensity levels ranging from 45 dB to 100 dB for different subjects within a given fold. Each model, denoted as M_x, is the model that has not seen subject x during training. The rows correspond to the scores obtained when the model is evaluated using a particular subject, providing a detailed performance metric for each individual.

Key observations from Table II reveal insightful patterns in the performance of different models across subjects. Each model, denoted as M_x where x ranges from 1 to 7, was trained by leaving out data from one specific subject during each iteration of cross-validation. The table presents average Mean Squared Error (MSE) scores along with their standard deviations across stimulus intensity levels from 45 dB to 100 dB for each subject-model pair. The models exhibit relatively consistent performance with minor variations. Model M_6 consistently shows slightly better performance compared to others, indicated by lower average MSE scores across subjects. For instance, it achieves an average MSE of 0.01 (with a standard deviation of 0.09) for Subject 1 and 0.37 (with a standard deviation of 0.18) for Subject 7. This suggests that M_6 effectively generalizes well to unseen data, possibly due to the characteristics of the training data it was exposed to. Model M_1, while not consistently the best performer across all subjects, shows notable strength in predicting loudness growth curves for Subject 1, with an MSE of 0.06 (standard deviation 0.19). This performance makes it a promising candidate for further evaluation, especially considering its effectiveness in predicting unseen data for Subject 1.

Overall, the table underscores the robustness of the models in predicting loudness growth curves across different subjects, highlighting the importance of cross-validation in evaluating model performance. The standard deviations provide insights into the variability of predictions, further aiding in understanding the reliability of each model's performance metrics. These findings support the selection of Model M_1 for subsequent analyses and comparisons against existing methodologies, ensuring rigorous evaluation and validation of our approach.

Table III compares the Mean Squared Error (MSE) scores

TABLE II: MSE scores averaged across all loudness levels and test cases (Standard deviations) for 7 models and for 7 subjects

Subject	M_1	M_2	M_3	M_4	M_5	M_6	M_7
1	0.06 (0.19)	0.02 (0.13)	0.03 (0.15)	0.02 (0.15)	0.02 (0.14)	0.01 (0.09)	0.01 (0.11)
2	0.28 (0.05)	0.41 (0.07)	0.37 (0.05)	0.26 (0.05)	0.23 (0.05)	0.27 (0.11)	0.16 (0.09)
3	0.15 (0.14)	0.24 (0.10)	0.22 (0.14)	0.14 (0.14)	0.12 (0.13)	0.12 (0.08)	0.06 (0.09)
4	0.06 (0.24)	0.04 (0.20)	0.05 (0.30)	0.10 (0.30)	0.07 (0.24)	0.04 (0.18)	0.08 (0.21)
5	0.09 (0.24)	0.05 (0.19)	0.06 (0.22)	0.09 (0.25)	0.27 (0.18)	0.08 (0.23)	0.12 (0.27)
6	0.06 (0.24)	0.07 (0.27)	0.06 (0.24)	0.06 (0.23)	0.07 (0.24)	0.11 (0.32)	0.13 (0.29)
7	0.33 (0.12)	0.22 (0.14)	0.25 (0.12)	0.34 (0.11)	0.39 (0.12)	0.37 (0.18)	0.51 (0.17)

TABLE III: Comparison of MSE scores averaged across all loudness levels and test cases for Model M_1 and method in [6], [7]

Models	1	2	3	4	5	6	7
M_1	0.06	0.28	0.15	0.06	0.09	0.06	0.33
Silva and Epstein [6], [7]	0.19	0.03	0.08	0.08	0.15	0.29	0.11

TABLE IV: Frechet Distance Obtained for All Subjects for Model M_1

Models	1	2	3	4	5	6	7
M_1	0.37	0.75	0.62	0.61	0.83	0.53	0.98

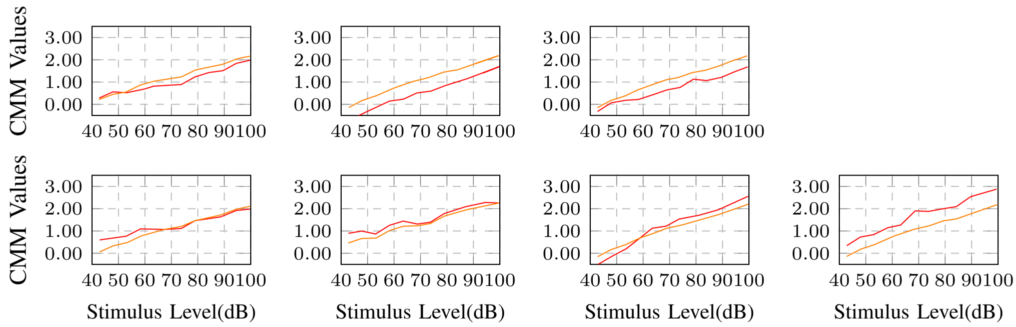


Fig. 3: Comparison of predicted (orange) and ground truth (red) loudness growth curve for seven subjects using model M_1.

averaged across all loudness levels and test cases for Model M_1 with those reported by Silva and Epstein [6], [7]. The table illustrates how Model M_1, selected based on its performance in predicting loudness growth curves for subjects at a stimulus frequency of 1 kHz, compares against established methodologies. Each entry in the table represents the MSE score for a specific subject, ranging from 1 to 7, indicating the predictive accuracy of Model M_1 relative to the referenced models.

Key observations reveal that Model M_1 consistently achieves competitive MSE scores across different subjects compared to Silva and Epstein's approach. Notably, Model M_1 exhibits strengths in predicting loudness growth curves for certain subjects, such as Subject 1 with an MSE of 0.06, demonstrating its efficacy in capturing variations in loudness perception across different stimuli intensities. These findings underscore the robustness of Model M_1 in predicting loudness growth curves and highlight its potential for advancing the understanding and application of predictive models in auditory perception research.

Table IV provides the Frechet distance scores calculated for Model M_1 across all seven subjects. The Frechet distance is a metric used to measure the similarity between two curves, typically comparing the predicted and ground truth loudness growth curves. A lower Frechet distance indicates a closer match between these curves. The scores in Table IV range from 0.37 to 0.98 across subjects 1 to 7, illustrating the varying degrees of accuracy in predicting loudness growth curves using Model M_1. Lower Frechet distances, such as 0.37 for Subject 1, indicate a closer match between the predicted and ground truth curves, suggesting that Model M_1 performs well in predicting loudness growth for some subjects but shows greater deviation for others.

To facilitate a thorough evaluation, we have generated visual representations of the predicted and ground truth loudness growth curves for all subjects using M1, presented in Figure 3. This figure depicts the relationship between CMM-value and intensity level (dB). In Subject 1, the predicted loudness growth curve closely mirrors the ground truth curve, indicative of model M_1's ability to accurately reproduce observed trends. This alignment is substantiated by a low Frechet distance of 0.37 (Table IV), underscoring the model's fidelity in predicting loudness responses in this specific case. Conversely, in Subject 7, discrepancies between the predicted and ground truth curves are more apparent, reflected in a higher Frechet distance of 0.98 (Table IV). Overall, the visual analyses presented in Figure 3 complement the quantitative metrics detailed in Tables II and IV, offering a comprehensive evaluation of model M_1's performance across a diverse cohort.

VI. Conclusion

Hearing loss represents a significant public health issue with profound implications for quality of life. In this study, we have introduced a novel approach employing deep learning to automate the estimation of loudness growth curves from raw Auditory Brainstem Response (ABR) signals, offering a valuable tool for hearing assessment. Our methodology encompasses several crucial steps: preprocessing of ABR signals and loudness growth data, feature extraction utilizing empirical wavelet transform with Fourier Bessel series expansion, and mapping these features to psychoacoustical estimates of loudness growth through multi-target regression.

The evaluation of our approach incorporates objective metrics such as Mean Squared Error (MSE) and Frechet distance, providing a rigorous quantitative assessment of our model's performance. Our experimental findings demonstrate the effectiveness of our method, showcasing competitive performance across diverse subjects and stimulus intensity levels. Notably, Model M_1 emerged as the top performer, particularly adept at predicting unseen loudness growth curves.

Comparative analysis with a prior study by Silva and Epstein underscores the favorable outcomes of our approach in terms of MSE and Frechet distance metrics. This highlights the robustness and reliability of our model in estimating loudness growth curves from ABR signals.

In summary, our study represents a significant advancement in hearing assessment techniques by introducing a robust and automated method for loudness growth curve estimation. This approach has the potential to deepen our understanding of auditory perception and facilitate the development of personalized solutions for individuals with varying degrees of hearing impairment. Future research directions should focus on validating and refining our model in populations with hearing impairments, thereby broadening its applicability and impact in clinical settings.

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