Temporal-Spatial Correlation Analysis for Ship-Radiated Noise Based on Random Matrix Theory

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***Abstract*—Time-domain methods for temporal and spatial correlation estimation often face challenges due to background noise and environmental factors, leading to low time delay resolution. This limitation reduces the accuracy of underwater acoustic source localization and depth estimation. To tackle this issue, this paper introduces a statistical approach to estimate temporal and spatial correlations in ship-radiated noise. Using random matrix theory (RMT), we employ a factor model to determine the number of dynamic factors and the binary correlation structure of residuals from ship-radiated noise. This technique effectively captures the essential characteristics of underwater acoustic signal dynamics. Our proposed RMT-based modeling approach provides a robust analysis of ship-radiated noise which exhibits a wide frequency band and non-Gaussian nature. Results show that this method is highly sensitive and reliable in detecting variations in the temporal and spatial correlation of ship-radiated noise. The proposed method provides a novel insight into improving temporal and spatial correlation estimation and enhancing situation awareness in underwater environments.**

I. INTRODUCTION

The underwater noise received by passive sonar includes ambient noise, ship-radiated noise from vessels (ships, submarines, et al.), and self-noise originating from the vessel machinery. Ship-radiated noise, in particular, is generated by the vibration of the propeller and various mechanical parts of the vessel during navigation. These vibrations radiate sound waves through the ship's hull, converting them into underwater acoustic waves. As ship-radiated noise propagates through the complex marine environment, it exhibits non-stationary, non-Gaussian, and nonlinear characteristics, which make it challenging for traditional Gaussian-based methods to analyze the underwater acoustic signal effectively. Therefore, this paper investigates a statistical method for the analysis of ship-radiated noise with a wide frequency band and non-Gaussian.

Temporal and spatial correlations in underwater noise arise from the pathways of propagation and various noise sources[1]. Temporal correlation primarily results from the multipath propagation of underwater radiated noise. Variations in the ocean's physical properties, such as temperature, salinity, and pressure, affect the propagation process through mechanisms like refraction, reflection, and scattering. Environmental factors such as water depth, seafloor topography, and marine life further influence noise propagation. Additionally, ocean surface phenomena like breaking waves, wind, and rain also impact the propagation of underwater noise[1][2]. When a vessel emits underwater signals, the scattered and direct waves create multiple propagation paths. As a result, sound waves reach the receiver at different times and intensities. The echoes from these propagation paths may overlap and mix, causing the received signal to differ from the original in terms of time delay and amplitude. Consequently, the multi-path effect significantly influences the temporal correlation of underwater acoustic signals. Additionally, the interaction between various mechanical noises from the vessel and background noise further affects the correlative characteristics of ship-radiated noise.

The spatial correlation of ship-radiated noise is primarily generated by mutual interference between hydrophones in the underwater noise field. Studies have shown that in an underwater spatially inhomogeneous and anisotropic noise field, refraction can significantly impact spatial correlation performance based on receiver locations. This is because ship noise sources are distributed near the surface and exhibit stratification[2][3]. The relative distance between receivers also affects the spatial correlation of underwater signals[3]. Another critical factor for the efficiency of the spatial correlation process is the stability of the receiver positions and the variability in receiver separation[3][4]. Correlation plays a crucial role in the study of underwater acoustic signals. By analyzing the spatial correlation matrix, researchers can assess the spectrum similarity measured at different times. Studies have demonstrated a certain degree of similarity between different frequency bands of underwater acoustic signals[2][5]. Spatial correlation is used for analyzing and estimating the travel time of acoustic signals[2][6]. Reference[7] employed spatial correlation to estimate the time Green's function. Additionally, reference[8] applied the spatial correlation technique to determine the ship noise spectrum and its modulation spectrum.

However, traditional methods of spatial correlation are primarily based on time-domain analysis. Due to the influence of complex background noise on underwater acoustic wave propagation, as well as the dependence of sound speed on temperature and seawater pressure, conventional methods for source localization and depth estimation typically rely on multi-path time delay and elevation angles obtained from hydrophone arrays. These time-domain analysis methods often result in low time delay resolution, thereby limiting the accuracy of the estimations.

To address this issue, we propose a modeling approach based on RMT. This method utilizes a factor model to analyze and construct the binary structure of residuals obtained from ship-radiated noise, revealing correlation variations in spatially inhomogeneous noise fields. This approach provides novel insights for underwater source localization and sonar detection. Implementing this method could potentially enhance the accuracy and reliability of these applications.

Recently, RMT has been a powerful mathematical tool for statistical data analysis and revealing correlations between observations. Such as detection techniques based on RMT have found applications in smart grids, intelligent healthcare, communications, and finance[9][10][11][12]. Its robust performance under low signal-to-noise ratio conditions has been well-documented in communications[13]. In recent years, RMT has also been applied to underwater acoustic signal research. These applications include separating predominant single-scattering from weak multiple-scattering of ultrasound in weakly inhomogeneous media, isolating loud directional sources from the isotropic diffuse noise field in ocean environments, and distinguishing interference noise components from diffuse noise in weakly inhomogeneous ocean environments[14][15][16]. RMT is also utilized for noise field modelings, such as modeling sound scattering by random sea bottom roughness and simulating sound propagation in randomly inhomogeneous oceanic noise fields[17][18]. Furthermore, RMT has been employed to detect events using Marchenko-Pastur and Tracy-Widom distributions with random acoustic signal noise[19].

RMT is a data-driven and statistical method that offers advantages over traditional time-frequency-based analysis schemes for underwater acoustic signals in the complex ocean noise environment. By introducing RMT into underwater acoustic signal processing, especially for studying correlations in inhomogeneous noise fields and identifying targets in low signal-to-noise ratio environments, researchers can improve algorithm accuracy and enhance capabilities in weak signal detection. This approach also increases the robustness of the system, providing new research perspectives for passive sonar signal detection technology.

This paper presents a statistical method based on RMT for ship-radiated noise and proposes a modeling approach to estimate temporal and spatial correlations. Utilizing a factor model based on RMT, it estimates the number of dynamic factors and constructs a binary correlation structure of residuals from ship-radiated noise[20]. The key contributions of this paper are summarized as follows:

(1)Propose a statistical approach for analyzing temporal and spatial correlations of ship-radiated noise based on RMT. This method is robust against interference noise and data errors.

(2)Introduce a modeling technique using a factor model to enhance the accuracy and reliability of ship-radiated noise analysis in non-Gaussian underwater noise environments.

(3)The proposed method is highly sensitive to variations in the temporal and spatial correlations of ship-radiated noise, making it valuable for target detection and enhancing situational awareness in underwater environments.

(4))Provide new insight into temporal and spatial correlations in ship-radiated noise by extracting dynamic factors and constructing the binary correlation structure for residuals.

The remaining part of the paper is organized as follows: Section II provides a detailed problem description. Section III introduces the proposed method, including the theoretical spectral distribution of residuals and temporal-spatial correlation modeling and estimation. Experimental validation is presented in Section IV, followed by conclusions in Section V.

II. Problem Description

Assuming there are M hydrophones deployed in a vertical line array underwater, consider the ship-radiated noise $h_i(t)$ received by i^{th} hydrophone. $h_i(t)$ consists of the superposition of $\mathrm{m}_j(t)$, the noise radiated from N ships, and n_i, the uncorrelated ambient noise[8]. Thus, the ship-radiated noise received from i^{th} hydrophone can be represented as:

$$h_i(t) = n_i(t) + \sum_{j=1}^{N} \alpha_{ij} \mathrm{m}_j(t - \tau_{ij}) \tag{1}$$

Where α_{ij} represents the attenuation parameter, and τ_{ij} is propagation time from ship j to the hydrophone i, $\mathrm{i} \in 1 \ldots \mathrm{M}$, $\mathrm{j} \in 1 \ldots \mathrm{N}$.

Assume $h_i(t)$ exists temporal and spatial correlation in an inhomogeneous underwater environment. Consider two hydrophones separated by a distance L, from which signals $\mathrm{h}_u(t)$ and $\mathrm{h}_v(t)$ are collected from u^{th} and v^{th} hydrophones. The temporal correlation can be obtained as:

$$\mathrm{R}_{uu}(\tau) = \int_{-\infty}^{+\infty} \mathrm{h}_u(\mathrm{t}) \mathrm{h}_u(t - \tau) dt \tag{2}$$

Spatial correlation mainly results from mutual interference between two or more hydrophones in the underwater noise field. The spatial correlation R_{uv} of the underwater acoustic signals h_u and h_v is defined as:

$$\mathrm{R}_{uv}(\tau) = \int_{-\infty}^{+\infty} \mathrm{h}_u(\mathrm{t}) \mathrm{h}_v(t - \tau) dt \tag{3}$$

III. The Proposed Method

Traditional RMT-based studies for ship-radiated noise analysis typically assume Gaussian characteristics in homogeneous underwater noise fields. However, in spatially inhomogeneous noise fields, the noise exhibits non-stationary, non-Gaussian, and nonlinear characteristics, making it difficult for traditional Gaussian-based methods to effectively analyze the underwater acoustic signals. In this section, a factor model is applied to capture the temporal-spatial correlation structure of ship-radiated noise in inhomogeneous noise fields. Initially, the subspace of residuals is obtained by removing several factors from the data. Then, the model for temporal-spatial correlation is developed by deriving the theoretical spectral distribution of residuals. Finally, by minimizing the distance between the theoretical and empirical distributions, the temporal-spatial correlations of ship-radiated noise are determined.

A. Residual Formulation for Ship-Radiated Noise

The ship-radiated noise consists of mechanical noise, propeller noise, and hydrodynamic noise. Mechanical noise and parts of propeller have low-frequency line spectrum components that are periodic and stable. These components can be extracted using several principal components. Thus, the ship-radiated noise can be decomposed into principal components and residuals. Assume matrix D represents ship-radiated noise recorded by the hydrophone array, which can be formulated as: $D = U + \sum_{p=1}^{n} L_p F_p$, where L_p is the matrix of factor loading, F_p is the matrix of factor, and p is the number of factors to be subtracted from the underwater acoustic signal. These factors reflect the spatial correlation of the underwater acoustic signals. When the first P most important factors are removed from the passive acoustic signal, the remaining components, referred to as residuals, U, are represented as follows:

$$U = D - \sum_{p=1}^{n} L_p F_p \tag{4}$$

The residuals contain irregular noise components made up of various sound sources, such as ocean environmental noise and part of hydrodynamic noise. Unlike traditional Gaussian noise, the residuals typically exhibit the characteristics of nonlinear colored noise, which include acoustic information about the ocean environment and rich ship-related knowledge. This information reflects the interaction between the ships and the surrounding marine environment, such as the impact of ship turbulence, acoustic properties of the ship surface material, as well as the size and shape of the ship. Therefore, an in-depth analysis of the residual components can reveal complex dynamics of the interaction between the ocean environment and ships.

B. Temporal-spatial Correlation Estimation

The next step is to model the covariance structure of residual components U using equation (4) and then estimate the temporal-spatial correlation. Let A_N and B_N be N × N and T × T symmetric non-negative definite matrices, respectively. The matrix A_N represents temporal correlation, while B_N represents spatial correlation[20]. The binary structure of the residue is then characterized by:

$$U = A_N^{1/2} \varepsilon {B_T}^{1/2} \tag{5}$$

Where $A_N = \left\{ (A_N)_{ii} = 1, (A_N)_{ij, i \neq j} = \eta, i, j = 1, \cdots N \right\}$, and $B_T = \{ (B_T)_\alpha = \exp(-|h-t|/\alpha, h, t = 1, \cdots T \}$, ε is a $N \times T$ matrix with independent identical distribution. Thus, the covariance matrix of the residual component U using equation(5) is obtained as:

$$C = \frac{1}{T} U U^T = \frac{1}{T} {A_N}^{\frac{1}{2}} \varepsilon B_T \varepsilon^T A^{\frac{1}{2}} \tag{6}$$

To simplify the model, two assumptions are made: first, we assume that total spatial correlations are efficiently removed from p principal components. So $\mathrm{A}_N \approx \mathrm{I}_{N \times N}$ or $\eta = 0$; second, the temporal correlation is exponentially decreasing, represented by$\{\mathrm{B}_T\} = b^{|i-j|}$, with $|b| < 1$.

Subsequently, the N transform of the covariance matrix C is obtained by utilizing the cyclic property of the trace and the FRV multiplication law as follows:

$$\begin{aligned}
&\mathrm{N}_{\frac{1}{T} A^{\frac{1}{2}} \varepsilon B \varepsilon^T A^{\frac{1}{2}}}(z) = N_{\frac{1}{T} A^{\frac{1}{2}} A^{\frac{1}{2}} \varepsilon B \varepsilon^T}(z) \\
&= N_{\frac{1}{T} A \varepsilon B \varepsilon^T}(z) (cyclic \quad property \quad of \quad trace) \\
&= \tfrac{z}{1+z} N_A(z) N_{\frac{1}{T} \varepsilon B \varepsilon^T}(z) (FRV \quad multiplication \quad law) \\
&= \tfrac{z}{1+z} N_A(z) N_{\frac{1}{T} \varepsilon^T \varepsilon B}(z) (cyclic \quad property \quad of \quad trace) \\
&= \tfrac{z}{1+z} N_A(z) \tfrac{rz}{1+rz} N_B(\mathrm{rz}) N_{\frac{1}{T} \varepsilon^T \varepsilon}(z) (FRV \quad multiplication \quad law) \\
&= \tfrac{z}{1+z} N_A(z) \tfrac{rz}{1+rz} N_B(\mathrm{rz}) \tfrac{(1+z)(1+rz)}{z} \\
&= rz N_A(z) N_B(\mathrm{rz})
\end{aligned} \tag{7}$$

Where $N_{\frac{1}{T} \varepsilon^T \varepsilon}(z) = \frac{(1+z)(1+rz)}{z}$. By taking $M \equiv M(z)$ and $z = M = M(z)$ derived from N transformation $M_X(N_X(z)) = N_H(M_X(z)) = z$, the above equation (7) can be reformulated as follows:

$$\mathrm{N}_{\frac{1}{T} A^{\frac{1}{2}} \varepsilon B \varepsilon^T A^{\frac{1}{2}}}(M(z)) = rM N_A(M) N_B(\mathrm{rM}) = Z \tag{8}$$

Assuming that the spatial correlation of residual is eliminated, the spatial correlation matrix is satisfied $A = I_N$. Consequently, $N_A(z) = N_I(z) = 1 + 1/z$. Therefore, the equation (8) is conveniently reformulated as follows:

$$\mathrm{rM} = \mathrm{M}_B\left(\frac{z}{r(1+M)}\right) \tag{9}$$

Next, we need to determine M_B for equation (9). The Temporal covariance matrix B from (6) follows AR(1) process, which can be represented in the simple form $B = b^{|i-t|}$. By applying a Fourier transform to matrix B, the M transform of the matrix B can be obtained:

$$M_B(z) = -\frac{1}{\sqrt{1-z} \sqrt{1 - \frac{(1+b^2)^2}{1-b^2} z}} \tag{10}$$

where $a^2 = 1 - b^2$. Finally, a fourth-order polynomial is obtained as follows:

$$\begin{aligned}
&a^4 c^2 M^4 + 2a^2 c(-(1+b^2)z + a^2 c) M^3 + ((1-b^2)^2 z^2 \\
&-2a^2 c(1+b^2) + (c^2-1) a^4) M^2 - 2a^4 M - a^4 = 0
\end{aligned} \tag{11}$$

Where $|b| < 1$, $a = \sqrt{1-b^2}$ and $c = \frac{N}{T}$.

To further determine the green's function of the binary structure model, it can be derived from the moment generating function $M(z)$ as follows:

$$G^{Model}(z) = \frac{M(z)+1}{z} \tag{12}$$

Finally, the spectral distribution of the covariance matrix given by equation(6), is estimated from the imaginary part of $G^{Model}(z)$:

$$\mathrm{H}^{Model}(b) = -\frac{1}{\pi}\lim_{\varepsilon\to 0^+} \Im m G(\lambda + i\varepsilon) \tag{13}$$

C. The Spectral Distribution Divergence

Assuming the underwater ambient noise remains relatively stable under normal conditions, we can use this typical ambient noise as a reference. The first step involves estimating an appropriate reference spectral distribution, denoted as H^{Model}, using the processes described in parts A and B. Next, we obtain the estimated spectral distribution, H^{Est} using online data. We then compare H^{Est} and H^{Model} by employing the Kullback-Leibler Divergence spectral distance as:

$$\begin{aligned} &D_{KL}(H^{Est}||H^{Model}) = \tfrac{1}{2}\sum_i H_i^{Est}\log\frac{H_i^{Est}}{Q_i} \\ &+\tfrac{1}{2}\sum_i H_i^{Model}\log\frac{H_i^{Model}}{Q_i} \end{aligned} \tag{14}$$

Where $Q_i = \frac{H_i^{Est}+H_i^{Model}}{2}$. The number of factors p and b approximate their true values when the divergence between the estimated and reference spectral distribution is sufficiently small. Here, p represents the spatial correlation parameter, and b represents the temporal correlation parameter.

IV. Experiment Validation

This section presents numerical results to illustrate the behavior of the temporal and spatial correlation in ship-radiated noise. Experiments are conducted to verify the performance of the proposed method.

A. Datasets

The dataset was collected in the South Sea of China between August 20 and 22, 2023. It was recorded using a vertical array of 64 hydrophones arranged in a line, as shown in Fig. 1, with a sampling rate of 32,000Hz. The hydrophone array is placed at a depth of approximately 400 meters. Data from the 64-channel array hydrophone was digitized and transmitted via a cable to a monitoring center for real-time processing and archival storage. The hydrophones were spaced approximately 6 meters apart along the vertical line. Then the original data underwent data cleaning and filtering processes. As a result, we obtained 28 hours of relatively pure ambient noise data as well as 40 hours of ship-radiated noise. In this section, we analyzed the correlation variation to gain a better understanding of the dynamic characteristics of underwater acoustic signals.

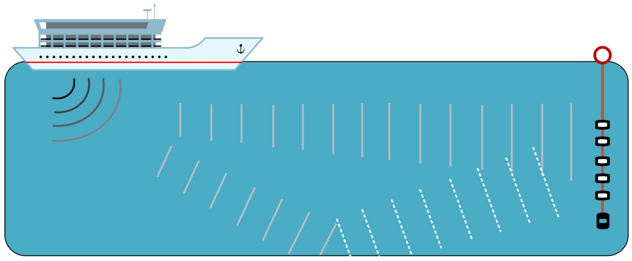

Fig. 1. Hydrophone setup for capturing underwater ship-radiated noise

B. Eigenvalue Distribution Analysis for Ship-Radiated Noise

In this subsection, we conducted experiments to validate the effectiveness of the proposed method for ship-radiated noise. Through detailed analysis and comparison of the eigenvalue distributions of the covariance matrix of residuals, we examined the statistical characteristics of the underwater acoustic signal and performed environmental perception. Fig. 2 illustrates the eigenvalue distribution before the ship entered the monitoring area. The window sizes are set to be 64x360. In contrast, Fig. 3 shows the eigenvalue distribution when the ship appeared in the monitoring area. The significant change in the eigenvalue distribution before and after the ship's appearance is evident. Therefore, the factor model effectively captures the statistical characteristics of ship-radiated noise in both states: before the ship's appearance and when a ship is present. The results indicate that during short-term events, such as ship appears, the eigenvalue distribution of the covariance matrix of residuals shows a sensitive response. This sensitivity enables the proposed method to detect variations in the ocean environment by analyzing the eigenvalue spectrum of the underwater acoustic signal.

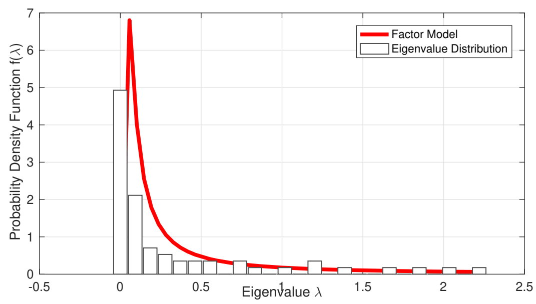

Fig. 2. Eigenvalue distribution before the ship's appearance.

C. Temporal and Spatial Correlation Estimation

In this subsection, we examine the temporal and spatial correlations in three scenarios: before, during, and after the ship's appearance. We also estimate the temporal and spatial correlations in those three states, represented by the parameters b and p. In this experiment, we used a sliding window method to process the data with a window size of 64x320. First, we estimated the spatial correlation coefficient P at different times,

REFERENCES

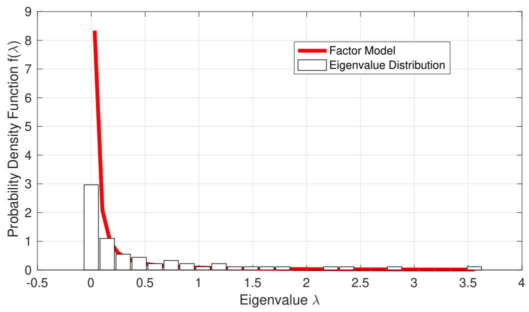

Fig. 3. Eigenvalue distribution when the ship appears.

as depicted in Fig. 4. The results indicate that spatial correlation parameter p increases when a ship is present at t=90 seconds. It is observed that p reflected spatial correlation and varies with different states of the underwater environment. As shown in Fig. 4, we compare the eigenvalue distribution of different states: before, during, and after the appearance of a ship. The findings demonstrate that the presence of the ship leads to an increase in spatial correlation and changes in the statistical characteristics of the data. This reveals that the proposed model can effectively capture the dynamic of the underwater acoustic signal.

In addition, we analyze the temporal correlation in different scenarios to reveal the characteristics of underwater acoustic signal dynamics, as shown in Fig. 5. A ship was present in the monitoring area at 1:17 AM on August 21, 2023. It is observed that the temporal correlation fluctuates with the appearance of the ship in the monitoring area. The temporal correlation parameter b is calculated for each set of correlation parameters and then averaged. The averaged values of b before (I), during (II), and after (III) the appearance of a ship are $\widehat{b}$=0.6644, 0.7942, and 0.6842, respectively, shown in Fig.5. These results demonstrate that the temporal correlation significantly increases when a ship is present in the monitoring area. This indicates that the temporal correlation parameter b is sensitive to changes in different states of the ocean environment.

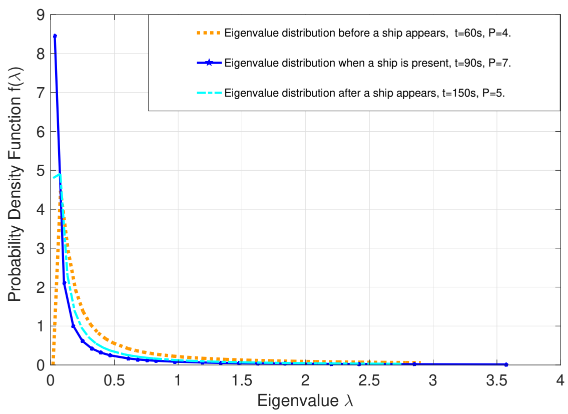

Fig. 4. Eigenvalue distribution and spatial correlation estimation.

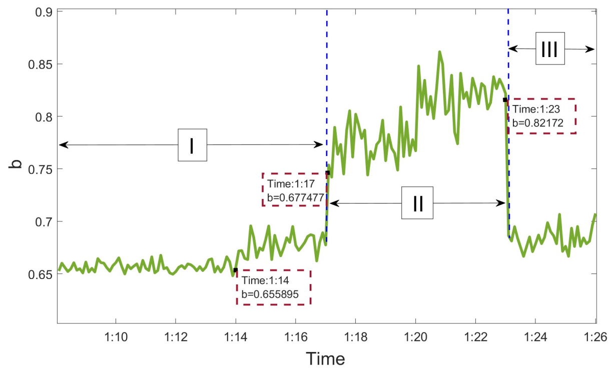

Fig. 5. Temporal correlation estimation for ship-radiated noise.

V. Conclusion

This paper introduces a new statistical approach using RMT to estimate the temporal and spatial correlations in ship-radiated noise. It proposes a novel method for modeling ship-radiated noise using a factor model. The results demonstrate that the method effectively captures variations in temporal and spatial correlation of ship-radiated noise. By monitoring these correlations across different scenarios, we gain insights into the dynamic characteristics of underwater acoustic signals. This approach offers new perspectives for monitoring water areas and improving marine situational awareness. Future research should focus on evaluating how well subsequent pattern recognition systems can utilize the eigenvalue distribution characteristic derived from RMT as a feature for analyzing underwater acoustic signals.

References

[1] S. H. Kim, B. K. Choi, and B.-N. Kim, "Correlation between underwater noise and sea level at ieodo ocean research station," *Journal of Marine Science and Engineering*, vol. 9, no. 1, p. 1, 2020.

[2] M. J. Buckingham, "Cross-correlation in band-limited ocean ambient noise fields," *The Journal of the Acoustical Society of America*, vol. 131, no. 4, pp. 2643–2657, 2012.

[3] E. Skarsoulis and B. Cornuelle, "Cross-correlation of shipping noise: Refraction and receiver-motion effects," *The Journal of the Acoustical Society of America*, vol. 145, no. 5, pp. 3003–3010, 2019.

[4] B. Yang, X. Zhang, Y. Hou, P. Hu, and K. Wu, "Simulation research of ocean environment noise spatial correlation," in *2019 IEEE 3rd Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC)*, IEEE, 2019, pp. 1723–1727.

[5] S. M. Nichols and D. L. Bradley, "Use of noise correlation matrices to interpret ocean ambient noise," *The Journal of the Acoustical Society of America*, vol. 145, no. 4, pp. 2337–2349, 2019.

[6] C. Jin, F. Ye, H. Zhang, and X. Bao, "Travel time picking of ambient noise cross-correlation using a deep neural network combining convolutional neural networks and transformer," *Acta Geophysica*, vol. 72, no. 1, pp. 97–114, 2024.

[7] J. Zhou, "Analysis of ambient noise spectrum level correlation characteristics in the china sea," *IEEE Access*, vol. 8, pp. 7217–7226, 2019.

[8] L. Fillinger, A. Sutin, and A. Sedunov, "Acoustic ship signature measurements by cross-correlation method," *The Journal of the Acoustical Society of America*, vol. 129, no. 2, pp. 774–778, 2011.

[9] R. Qiu, L. Chu, X. He, Z. Ling, and H. Liu, "Spatio-temporal big data analysis for smart grids based on random matrix theory: A comprehensive study," *arXiv preprint arXiv:1708.04935*, 2017.

[10] P. Sarma and S. Barma, "Emotion recognition by distinguishing appropriate eeg segments based on random matrix theory," *Biomedical Signal Processing and Control*, vol. 70, p. 102 991, 2021.

[11] R. Qiu and M. Wicks, *Cognitive networked sensing and big data*. Springer, 2014.

[12] Q. Feng, G. Radman, and X. Li, "Early anomaly detection for power systems based on kullback-leibler divergence using factor model analysis," *American Journal of Electrical Power and Energy Systems*, vol. 10, no. 4, pp. 60–73, 2021.

[13] L. Zheng, R. C. Qiu, Q. Feng, and X. Li, "Shifting maximum eigenvalue detection in low snr environment," *arXiv preprint arXiv:1802.10325*, 2018.

[14] R. Menon, P. Gerstoft, and W. S. Hodgkiss, "Cross-correlations of diffuse noise in an ocean environment using eigenvalue based statistical inference," *The Journal of the Acoustical Society of America*, vol. 132, no. 5, pp. 3213–3224, 2012.

[15] A. Aubry and A. Derode, "Multiple scattering of ultrasound in weakly inhomogeneous media: Application to human soft tissues," *The Journal of the Acoustical Society of America*, vol. 129, no. 1, pp. 225–233, 2011.

[16] G. Li, J. Liu, and S. Zhang, "Enhancing cross correlations of ocean ambient noise in the time domain based on random matrix theory," *The Journal of the Acoustical Society of America*, vol. 152, no. 5, pp. 2849–2858, 2022.

[17] D. V. Makarov, P. S. Petrov, and M. Y. Uleysky, "Random matrix theory for sound propagation in a shallow-water acoustic waveguide with sea bottom roughness," *Journal of Marine Science and Engineering*, vol. 11, no. 10, p. 1987, 2023.

[18] D. Makarov, "Modeling of sound propagation in the ocean by means of random matrices," in *2017 Days on Diffraction (DD)*, IEEE, 2017, pp. 227–232.

[19] B. A. E. Bencharif, I. Ölçer, E. Özkan, B. Cesur, *et al.*, "Detection of acoustic signals from distributed acoustic sensor data with random matrix theory and their classification using machine learning," in *SPIE Future Sensing Technologies*, SPIE, vol. 11525, 2020, pp. 389–395.

[20] J. Yeo and G. Papanicolaou, "Random matrix approach to estimation of high-dimensional factor models," *arXiv preprint arXiv:1611.05571*, 2016.