A Two-Stage Wall Parameters Estimation Algorithm for MIMO Through-the-Wall Radar

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***Abstract*—This paper proposes a two-stage wall parameter estimation method based on multi-channel to retrieve the unknown position, thickness, and relative permittivity of wall. In the first stage, it aims to estimate the slope and intercept of the front surface of the wall. This is achieved by formulating a cost-minimization problem, where the position of the front wall is modeled using a linear analytical expression. Subsequently, in the second stage, it focuses on calculating the positions of equivalent arrays that are aligned parallel to the wall for each transmitting antenna, so as to enable the estimation of the wall's thickness and its relative permittivity. The effectiveness of the algorithm is validated through electromagnetic simulations.**

***Index Terms*—Two-stage wall parameter estimation, through-the-wall imaging, equivalent array position approximation, linear MIMO array**

I. Introduction

Through-the-wall imaging radar (TWIR) uses electromagnetic (EM) waves to penetrate building walls for concealed target detection, localization, tracking and imaging. It has significant applications in counter-terrorism, surveillance and disaster rescue. During the imaging process, neglecting wall penetration effects lead to defocused target images and positional shifts, posing difficulties in subsequent target localization and identification [1]–[3]. Therefore, the precise estimation of wall parameters is crucial for effective TWIR.

Existing methods for estimating unknown wall parameters fall into two categories: wall feature value matching-based and time-delay-based methods. The former feature value based methods involve extracting parameters from radar echoes and matching them with known wall parameters. For example, in [4], matching is achieved by altering the array or its position relative to the wall. However, this method is operationally complex and constrained by the site. The time-delay methods estimate wall parameters based on the delay difference between echoes from the front and back walls, and they involve using various array structures or adjusting the distance from the array to the wall [5], including algorithms based on the eigenvalue decomposition of correlation matrices [6] and sparse reconstruction-based estimation algorithms [7]. The accuracy of these methods depends on the time-delay estimation results, and require the array structure to be parallel to the wall, which may limit its further application. This paper introduces a two-stage algorithm to estimate unknown parameters of inclined walls using a multiple-input-multiple-out (MIMO) array. In the initial stage, the algorithm iterates through various slopes and intercepts corresponding to the wall's geometry. During this process, it identifies the mirror points that correlate with the position of the transmitting antenna. Subsequently, a cost function is constructed to determine the set of slope and intercept values that most closely approximate the actual parameters of the wall. In the second stage, it calculates the equivalent array positions parallel to the wall, so as to estimate the thickness and relative permittivity of the wall. EM simulation validates the effectiveness of the proposed algorithm.

II. The Propagation of EM Waves in a Single Layer Wall

As shown in Fig. 1, the transmit antenna *(TX)* and receive antenna *(RX)* are horizontally placed. Assuming that the wall is homogeneous and smooth, the position of the front wall is expressed by equation $y = kx+b$. The line parallel to the front wall passing through *TX* is labeled as $y_2 = kx + c_1$, with h representing the distance from *TX* to the front wall. The mirror point of *TX* with respect to the front wall is *TX'*. Connecting *TX'* to *RX*, the intersection points at *y* and y_2 are defined as P_1 and *Q*, respectively. The angle between the line of the transmitting and receiving antennas and *y* is θ. The thickness and relative permittivity of the wall are denoted as d and ε, respectively. The red solid line represents the reflection path of EM on the front surface of the wall. The propagation path of EM on the rear surface of the wall is $TX \to P_2 \to P_3 \to P_4 \to RX$, intersecting y_2 at point *D*, whose coordinate is (x_D, y_D). *D* serves as the virtual equivalent receiving antenna for *RX* on this parallel line.

III. Estimation of Wall Parameters

A. Estimation of the Front Wall Position

Defining the coordinate origin as o, for the m-th (m=1,2,...,M) transmitting antenna and the n-th (n=1,2,...,N) receiving antenna, their coordinates are defined as $TX_m\,(x_m, y_m)$ and $RX_n\,(x_{rn}, y_{rn})$, respectively. The symmetric point of the m-th transmitting antenna with respect to the front wall is TX'_m, which is denoted as $({x_m}', {y_m}')$, where

$$ {x_m}' = (2k \cdot (y_m - b) - x_m(k^2 - 1))/(k^2 + 1) \tag{1} $$

$$ {y_m}' = (2 \cdot (b + kx_m) + y_m(k^2 - 1))/(k^2 + 1) \tag{2} $$

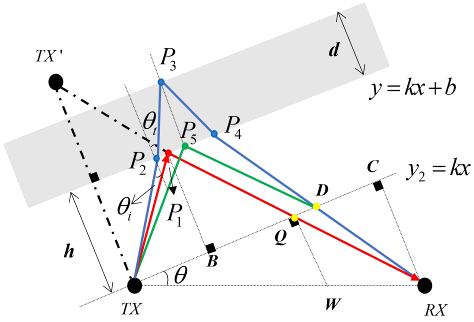

Fig. 1. Electromagnetic wave paths

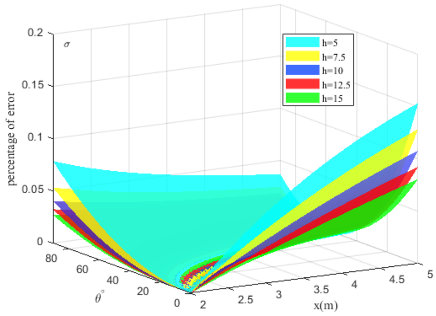

Fig. 2. Error percentage of (12)

The actual reflection path of the EM wave with respect to the front wall is denoted as $TX_m \rightarrow P_1 \rightarrow RX_n$, which can be equivalently represented by the propagation from the symmetric point $TX_m{}'$ to RX_n. The real and estimated time delay from TX to RX can be defined as t_n and $\hat{t}_n$. Specifically, $\hat{t}_n$ can be expressed as

$$\hat{t}_n(k,b) = \frac{\left\|TX_m{}'(k,b) - RX_n\right\|}{c} \tag{3}$$

where *c* represents the speed of light, $\|\cdot\|$ represents Euclidean distance. $TX_m{}'(k,b)$ represents the position of $TX_m{}'$ with different *k* and *b*.

The mean square error between t_n and $\hat{t}_n$ is defined as the cost function for estimating *k* and *b*. The parameters *k* and *b* can be estimated by minimizing the following cost function using Nelder-Mead algorithm [8].

$$f(k,b) = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}\left|t_n - \hat{t}_n(k,b)\right|^2 \tag{4}$$

where $f(k,b)$ represents the amplitude of the function above with different k and b. The fundamental principle of this algorithm lies in the assumption that minimizing the amplitude of the cost function results in the theoretical time delay closely matching the measured time delay. This alignment is crucial as it enables the accurate estimation of the values for *k* and *b*, which are key parameters in the algorithm's process.

B. Estimation of Thickness and Permittivity of the Wall

For the model depicted in Fig. 1, the reception at *RX* is primarily composed of three main parts: direct coupling between *TX* and *RX*, reflection on the front surface of the wall (NSR), and reflection on the rear surface of the wall (FSR). We use *l* to denote electrical lengths. All lengths mentioned below refer to electrical lengths. The length of NSR can be expressed as

$$l_{NSR} = l_{TXP_1} + l_{P_1RX} = 2l_{TXP_1} + l_{QRX} \tag{5}$$

Refraction occurs at point P_2, where the incident angle θ_i and refracted angle θ_t follow Snell's Law and can be expressed as

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\varepsilon} \tag{6}$$

where ε represents the relative permittivity of the wall. The equivalent electrical length of FSR can be expressed as

$$l_{FSR} = 2(l_{TXP_2} + \sqrt{\varepsilon}l_{P_2P_3}) + l_{DRX} \tag{7}$$

According to geometric relationships, we can obtain

$$l_{TXP_2} = l_{TXP_5} - d\tan\theta_t\sin\theta_i \tag{8}$$

$$l_{P_2P_3} = d/\cos\theta_t \tag{9}$$

Furthermore, we can derived the following equation by putting (8) and (9) into (7)

$$l_{FSR} = 2(l_{TXP_5} + d\sqrt{\varepsilon - \sin^2\theta_i}) + l_{DRX} \tag{10}$$

The time delay difference between the front and rear surfaces is the function of the incident angle for ε and *d*.

$$\begin{aligned} t_d(\theta_i) &= (l_{FSR} - l_{NSR})/c \\ &= \left(2\left(l_{TXP_5} - l_{TXP_1} + d\sqrt{\varepsilon - \sin^2\theta_i}\right) + l_{DRX} - l_{QRX}\right)/c \\ &= \left((l_{TX'D} + l_{DRX}) - l_{TX'RX} + 2d\sqrt{\varepsilon - \sin^2\theta_i}\right)/c \end{aligned} \tag{11}$$

Assuming the mirror point of the transmitting antenna is sufficiently far from the receiving antenna, we have

$$l_{TX'RX} - l_{TX'D} \approx l_{DRX} \tag{12}$$

As shown in Fig. 1, the distance between *TX* and *RX* is defined as *x*. Keeping the position of the equivalent receiving antenna *D* unchanged, place *RX* at any point on the extension line of P_4D. As the position of *RX* varies, *θ*, *x*, and the position of *Q* also vary accordingly. The error percentage of (12) can be represented by

$$\sigma = \left|\frac{l_{TX'D} + l_{DRX} - l_{TX'RX}}{l_{TX'D} + l_{DRX}}\right| \tag{13}$$

Fig. 2 illustrates the percentage of error at different $\theta\,(0^\circ \sim 90^\circ)$ and $x\,(l_{TXD} \leq x \leq h_{\min})$ as *h* varies. When the distance from *TX* to the front wall is 1.5m, the error rate doesn't exceed 15.3%. Furthermore, as *TX* moves farther from the front wall, the error percentage decreases. At this point, $l_{TX'D}+l_{DRX}$ approximates a straight line, implying that point *D* can be approximated as the point *Q*.

For a MIMO array with M transmitting elements and N receiving elements, building upon (11), the time delay between the first reflection and the second reflection for the m-th transmitting element and the n-th receiving element is defined as $t_d(m,n)$, which is given by

$$t_d\left(m,n\right) \approx \left(2d\sqrt{\varepsilon - \sin^2\theta_i(m,n)}\right)/c \tag{14}$$

where

$$\theta_i\left(m,n\right) \approx \arctan\left(\left|x_Q - x_m\right|/2h\right) \tag{15}$$

The line through the m-th TX parallel to the front wall is defined as $y_2^m = kx + c_1^m$, and the distance from TX_m to the front wall is expressed as h_m. Since k can be obtained from (4), by substituting the coordinate of the TX_m, we can obtain

$$c_1^m = y_m - kx_m \tag{16}$$

$$\theta = \arctan k \tag{17}$$

h_m can be expressed as

$$h_m = |y - y_2^m| \cos\theta = |b - c_1^m| \cos\theta \tag{18}$$

According to [11], the following equation is hold

$$0.25c^2{t_d}^2(m,n) = d^2\varepsilon - d^2\frac{\left(x_D - x_m\right)^2}{\left(x_D - x_m\right)^2 + 4{h_m}^2} \tag{19}$$

As shown in Fig. 1, the ratio of h and distance from RX to y_2 is defined as γ, which is given by

$$\gamma = \frac{h}{|RX - C|} \tag{20}$$

Since *TX'* is the mirror point of *TX*, it can be deduced that

$$|TX - TX'| = 2h = 2\gamma\,|RX - C| \tag{21}$$

The triangle *TX'* - *TX* - *Q* is similar to the triangle *Q* - *C* - *RX*, and the same as the triangle *TX* - *Q* - *W* to the triangle *TX* - *C* - *RX*. Based on the similarity of triangles, the relationships among their sides can be derived as follows:

$$\frac{|TX - TX'|}{|RX - C|} = \frac{|TX - Q|}{|Q - C|} \tag{22}$$

$$\frac{|TX - Q|}{|TX - C|} = \frac{|Q - W|}{|RX - C|} \tag{23}$$

Subsequently, the length of QW can be determined as

$$l_{QW} = \frac{2\gamma}{2\gamma + 1}\,|RX - y| \tag{24}$$

The coordinate on the Y-axis of point Q is

$$y_Q = y_m + l_{QW}\cos\theta = y_m + l_{QW}\cdot\arctan k \tag{25}$$

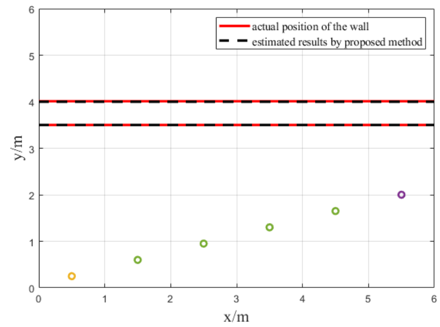

Fig. 3. Estimation results for wall position

Substituting (24)(25) into $y_2 = kx + c_1$, the coordinates of point Q can be determined as (x_Q, y_Q). Plugging these coordinates into (19) and integrating parameter values from each channel, we obtain

$$\mathbf{A} = \begin{bmatrix} 1 & -\frac{\left(x_Q^1 - x_1\right)^2}{\left(y_Q^1 - y_1\right)^2 + 4{h_1}^2} \\ 1 & -\frac{\left(x_Q^2 - x_1\right)^2}{\left(y_Q^2 - y_1\right)^2 + 4{h_1}^2} \\ \vdots & \vdots \\ 1 & -\frac{\left(x_Q^n - x_m\right)^2}{\left(y_Q^n - y_m\right)^2 + 4{h_m}^2} \end{bmatrix}_{MN\times 2} \tag{26}$$

$$\mathbf{p} = \begin{bmatrix} d^2\varepsilon \\ d^2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \tag{27}$$

where (x_Q^n, y_Q^n) $(n = 1, 2, \ldots, N)$ represents the position of the equivalent virtual receiving antenna for the n-th receiving antenna corresponding to each transmitting antenna.

In that case

$$\mathbf{Ap} - \mathbf{b} = \mathbf{0} \tag{28}$$

Therefore

$$\mathbf{p} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b} \tag{29}$$

$$\mathbf{b} = \begin{bmatrix} 0.25c^2{t_d}^2\left(1,1\right) \\ 0.25c^2{t_d}^2\left(1,2\right) \\ \vdots \\ 0.25c^2{t_d}^2\left(m,n\right) \end{bmatrix}_{MN\times 1} \tag{30}$$

$$\hat{d} = \sqrt{p_2} \tag{31}$$

$$\hat{\varepsilon} = p_1/p_2 \tag{32}$$

By minimizing the cost equation, we obtain d and ε.

$$f(d,\varepsilon) = |\mathbf{Ap} - \mathbf{b}|^2 \tag{33}$$

IV. Simulation Results and Analysis

To validate the effectiveness of the proposed algorithm, we deploy a UWB MIMO radar with a wall parallel to the ground, which has a relative permittivity of 6 and a thickness of 0.5m. The slope and intercept of the wall are 0 and 3.5, respectively. Using a linear array with two transmitters and

TABLE I
ESTIMATION RESULTS UNDER DIFFERENT SNRS

SNR/db	proposed algorithm		algorithm in [12]	
	d/m	ε	d/m	ε
30	0.515	6.05	0.523	5.863
25	0.517	6.06	0.523	5.86
20	0.52	6.08	0.527	5.793
15	0.523	5.89	0.53	5.6
10	0.53	5.84	0.533	5.6

four receivers, positioning in a straight inclined line. The transmitting antennas are located at coordinates (0.5m, 0.25m) and (5.5m, 2m). The simulation utilizes GprMax [9] to generate the echoes, where the ricker pulse with 1.5 GHz center frequency is set as the transmitting signal. In the estimation procedure, the searching space of slope and intercept are set to $k \in [-0.5, 0.5]$, $b \in [0, 5]$. After the first stage simulation, we can obtain the estimated results are $k = 0.0087$ and $b = 3.5$.

Next, using k and b as prior conditions to estimate thickness and relative permittivity of the wall. The search ranges are set to $d \in [0, 1]$, $\varepsilon \in [5.5, 8.5]$. The gaussian white noise is added to achieve a signal-to-noise ratio (SNR) of 30 dB. After the second stage stimulation, we can get the estimated wall thickness is 0.515m, and the relative permittivity is 6.05. The simulation results are depicted in Fig. 3, where the black line represents the estimated wall position.

In order to assess the algorithm's performance under different SNRs, we choose to quantitatively evaluate the estimated results using the error rate.

$$error = \frac{|\alpha_{real} - \alpha_{est}|}{\alpha_{real}} \times 100\% \tag{34}$$

where α_{real} is the true value of the parameter, and α_{est} is the estimated value of the parameter.

The error rates of thickness and relative permittivity for the proposed method compared with the method proposed in [12] at different SNRs are shown in Fig. 4(a) and Fig. 4(b), respectively. It's obvious that the method presented in this paper exhibits lower error rates under different SNRs. Table I presents the estimated parameters using both the method from reference [12] and the approach proposed in this paper under various SNRs. At a SNR of 10 dB, the estimation errors for wall thickness and relative permittivity are 0.03m and 0.16, respectively, which demonstrate the superiority of the algorithm.

V. CONCLUSION

This paper introduces a two-stage algorithm to estimate parameters of inclined walls. Initially, it employs mirror reflection for approximate calculations to find time delays between the front surface of the wall and potential parameters. Then, by minimizing a cost function, it accurately estimates the slope and intercept of the front wall, thus establishing the linear equation of the front wall. Using this equation, the linear array is transformed into a virtual array parallel to the wall for each channel. After combining all the channels, it enables the extraction of unknown wall thickness and relative permittivity. The method is compared with other approaches, and simulation results validate its robust performance.

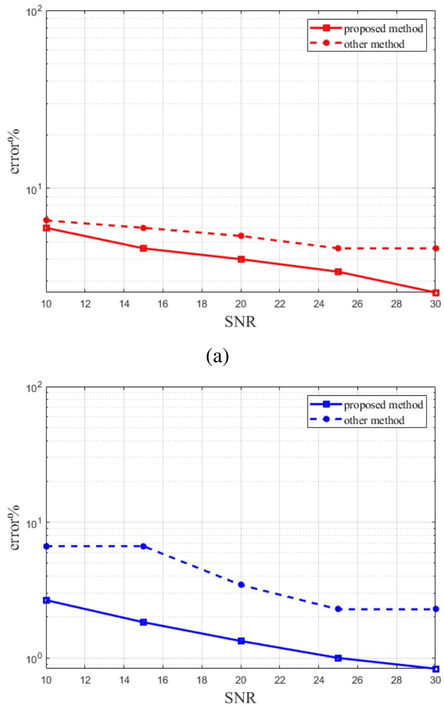

Fig. 4. Error rates of wall parameters for two methods at different SNRs (a) Wall thickness error rate (b) Relative permittivity error rate.

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